IT 50700 Measurement and Evaluation of Industry and Technology

Lecture Note (sections 4-1 to 4-7)

Chapter 4 Continuous Random Variables and Probability Distributions

Based on the text book: Applied Statistics and Probability for Engineers, 6th Ed, by D. C. Montegomery and G. C. Runger, published by Wiley

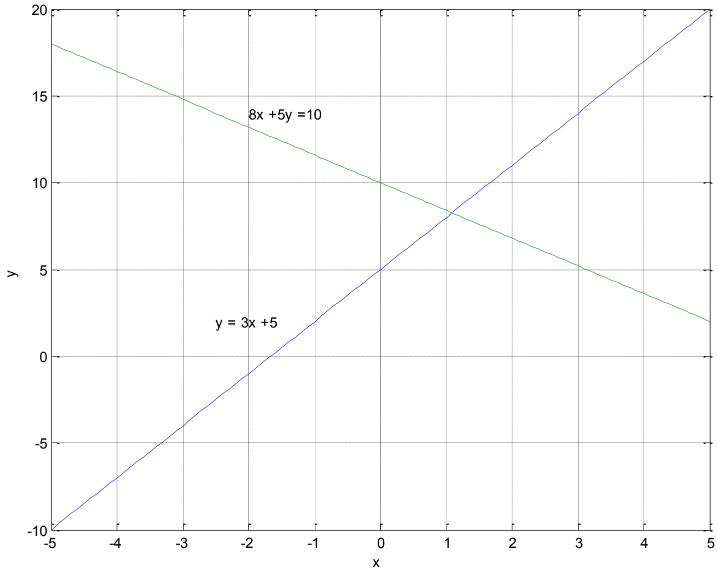
Math Review and MATLAB Solutions

Linear Equation

x = a ; vertical line through (a, b)
y = b; horizontal line through (
Y = mx +b, where m is the slope

Example
y = 3x + 5
8x + 5y = 10

MATLAB Script
```
% y = 3x + 5;  x = 0, y = 5
% 8x + 5y = 10; x = 0, y = 10
x = -5:0.1:5;
y1 = 3*x + 5;
y2 = -(8/5)*x + 10;
plot(x, y1, x, y2), grid on;
xlabel('x '); ylabel('y ');
text(-2.5,2, 'y = 3x +5');
text(-2,14, '8x +5y =10');
```



Function

$X^2 + y^2 = 1$

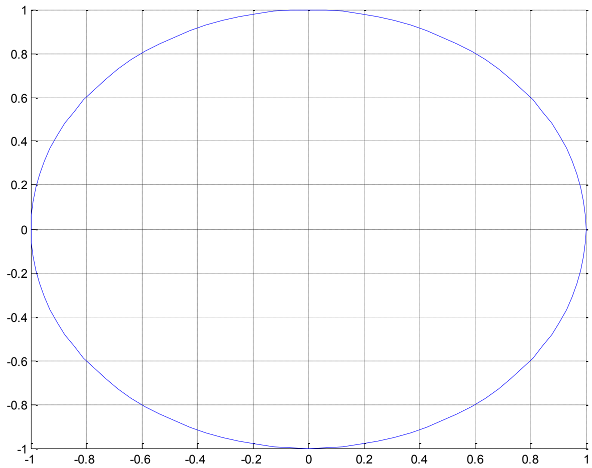
$y^2 = 1 – x^2$ or

$y = \pm \text{sqrt}(1 – x^2)$

```
function h = circle(x, y, r)
hold on
theta = 0:pi/50: 2*pi;
xunit = r *cos(theta) + x;
yunit = r *sin(theta) + y;
h = plot(xunit,yunit);grid on
hold off
```

Call the function circle to plot a unit circle with its center at (0,0), and a radius of 1.

circle(0,0,1)



Polynomial Functions and Their Derivatives

Reference: Calculus and Analytics Geometry, 6th Ed, by Thomas/Finney

Let y = f(x) define a function of. If the limit dy/dx exist and is finite, we call this limit the derivative of f at x and say that f is differentiable at x.

$$\frac{dy}{dx} = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

- The derivative of a constant is zero
- For positive integer powers of x: If n is a positive integer, then $\frac{d\,(x^n)}{dx} = nx^{n-1}$
- Constant multiples: If u = f(x) is any differentiable function of x, and c is a constant, then
  $$\frac{d\,(cu)}{dx} = c\frac{du}{dx}$$
- Positive integer powers of a differentiable functions: If u is a differentiable function of x and n is a positive integer, then u^n is differentiable and
  $$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$$
- The quotient rule: At a point where v ≠ 0, the quotient y = y/v of two differentiable functions is differentiable and
  $$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Examples:

Given the following polynomial functions:

$f(x) = x^3 + 2x + 4$

$h(x) = 6x^4$

$\emptyset(x) = 3$

$y = (x^2 - 3x + 5)^4$

The derivatives of f(x), h(x), and ϕ(x) are:

$f'(x) = 2x^2 + 2$

$h'(x) = 4 \cdot 6x^3 = 24x^3$

$\emptyset'(x) = 0$

$$\frac{dy}{dx} = 4(x^2 - 3x + 5)^3 \cdot \frac{d}{dx}(x^2 - 3x + 5) = 4(x^2 - 3x + 5)^3 \cdot (2x - 3) = 4(2x - 3)(x^2 - 3x + 5)^3$$

MATLAB Differentiate Symbolic Expression/Function,
http://www.mathworks.com/help/symbolic/diff.html

```
>> % diff_exs.m
syms x fx
fx = x^3 +2*x +4;
dfx = diff(fx, 1)
hx = 6*x^4;
dhx = diff(hx, 1)
dfx =
3*x^2 + 2
dhx =
24*x^3
```

The Fundamental of Integral Calculus, Reference: Calculus and Analytics Geometry, 6th Ed, by Thomas/Finney

Theorem: The First Fundamental Theorem of Integral Calculus

- If f is continuous on [a, b] and F is any anti-derivative of f on [a, b], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Examples

The area under the line y = mx:

$$\int_a^b mx\,dx = \frac{mx^2}{2}\Big]_a^b = \frac{mb^2}{2} - \frac{ma^2}{2}$$

The area under the curve y = x^3:

$$\int_a^b x^3\,dx = \frac{x^4}{4}\Big]_0^b = \frac{x^4}{4} - 0 = \frac{b^4}{4}$$

Calculate the area bounded by the x-axis and the parabola y = 6 – x – x^2.
Solution:
Find where the curves crosses the x axis:
y = 0, y = 6 – x – x^2 = (3 +x)(2-x) = 0 , which gives x = -3, or x = 2.

The area is

$$\int_{-3}^{2}(6 - x - x^2)dx = 6x - \frac{x^2}{2} - \frac{x^3}{3}\Big]_{-3}^{2} = \left(12 - \frac{4}{2} - \frac{8}{3}\right) - \left(-18 - \frac{9}{2} + \frac{27}{3}\right) = 20\frac{5}{6}$$

MATLAB Integration, http://www.mathworks.com/help/symbolic/integration.html

MATLAB int()

- Definite and indefinite Integral, http://www.mathworks.com/help/symbolic/int.html
- Indefinite Integral of Univariate Expression (Symbolic Integration):

The area under the line y = mx:

$$\int_a^b mx\,dx = \frac{mx^2}{2}\Big]_a^b = \frac{mb^2}{2} - \frac{ma^2}{2}$$

The area under the curve y = x^2:

$$\int_a^b x^3\,dx = \frac{x^4}{4}\Big]_0^b = \frac{x^4}{4} - 0 = \frac{b^4}{4}$$

MATLAB Solution:
Calculate the area bounded by the x-axis and the parabola y = 6 – x – x^2.
Find where the curves crosses the x axis:
y = 0, y = 6 – x – x^2 = (3 +x)(2-x)=0, which gives x = -3, or x = 2.

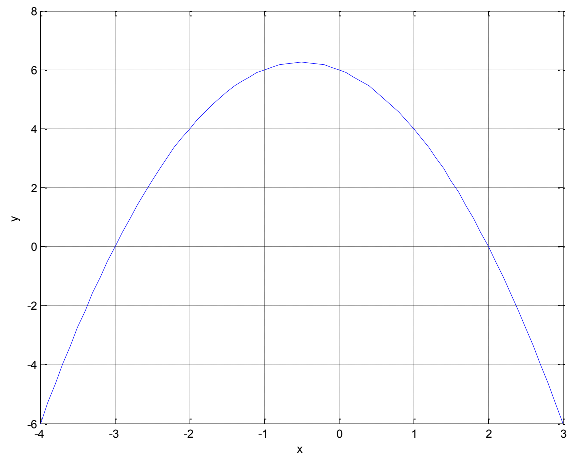
```
% intg_parabola.m
% Plot the parabola
x = -4:0.1:3;
fx = (6 - x -x.^2);
plot(x, fx), grid on;
xlabel('x'), ylabel('y');
```

```
syms x
m = 3;
y = int(m*x)
y1 = int(x^m)
y3 = int(6-x-x^2, -3, 2)
```

y = (3*x^2)/2

y1 = x^4/4

y3 = 125/6



Example 4-1 (page 110) and Example 4-6 (page 114)

Electric Current through a thin copper wire.

- X is the continuous RV of the current measured in a thin copper wire in mA.
- X is in the range of [4.9, 5.1] mA.
- f(x) = 5, the probability density function

Compute

$$P(X < 5) = \int_{4.9}^{5} f(x)dx = \int_{4.9}^{5} 5dx = 0.5$$

$$P(4.95 < X < 5.1) = \int_{4.95}^{5.1} f(x)dx = \int_{4.95}^{5.1} 5dx = 0.75$$

MATLAB Solution:

```
%ex4_6.m
% f(x) = 5
% E(X) = INTEGRAL(x*fx)dx; UL = 5.1, LL = 4.9
% E(X) = 5
% V(X) = INTEGRA(x - 5)^2 f(x) dx = 0.0033
syms x
% f =5;
F = 5*x;
```

```
EX = int(F, 4.9, 5.1);
VX = int(5*(x - EX)^2, 4.9, 5.1);
EX
VX
```

Run the ex4_6.m Script:

```
>> ex4_6
EX =  5
VX = 1/300
```

The Fundamental of Integral Calculus, Reference: Calculus and Analytics Geometry, 6th Ed, by Thomas/Finney

- **The Exponential Function e^x, pages 406-413**

e = 2.718281828459045 …
ln (e) = 1
ln (e^n)= n ln(e) = n

The function y = e^x
For every real number x, the number ex is defined to be ln-1 x:
$e^x = \ln^{-1} x$.
That is, y = e^x iff x = ln y

Examples:
$e^{\ln 2} = 2$
$e^{\ln(x^{\wedge} 2+1)}$ = x^2 + 1

- **The Derivative of y = e^x**

$$\frac{d}{dx}(e^x) = e^x$$

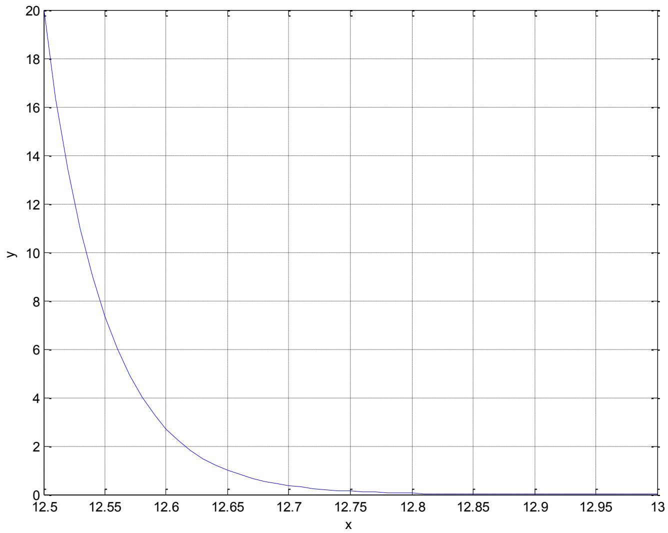
Example 4-2. Hole Diameter. Page 110.

- Let X denote the diameter of a hole drilled in a sheet metal component.
- The target diameter is 12.5 mm. Most random disturbances to the process result in larger diameters.
- Historical data show that the distribution of X can be modeled by a probability density function $f(x) = 20e^{-20(x-12.5)}, x \geq 12.5$

Q1. If a part with a diameter greater than 12.60 mm is scrapped, what proportion of parts is scrapped.

$$P(X > 12.60) = \int_{12.6}^{\infty} f(x)dx = \int_{12.6}^{\infty} 20e^{-20(x-12.5)}\,dx = -e^{-20(x-12.5)}\Big]_{12.6}^{\infty}$$

$$P(X > 12.60) = 0 + \left(\frac{1}{e^{20(x-12.5)}}\right) = \frac{1}{e^{20(0.1)}} = \frac{1}{e^2} = 0.135$$



Q2: What proportion of parts is between 12.5 and 12.6 mm?
Answer: P(12.5 < X < 12.6) = 0.865

```
% ex4_2.m
%
x = 12.5:0.01:13.0;
fx = 20*exp(-20*(x-12.5));
plot(x, fx), grid on;
xlabel('x'), ylabel('y');

syms x fx
fx = 20*exp(-20*(x-12.5));
PX = int(fx,12.5, 12.6) % P(12.5 < X < 12.6) = 1 - P(X > 12.6)
% PX = 1 - exp(-2) = 1 - /(exp(2) = 1 - 0.135 = 0.865
```

4.3 Cumulative Distribution Functions

The cumulative distribution function of a continuous RV X is

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du, \;\; for -\infty < x < \infty.$$

Ex. 4.3 Electric current. (Page 112)
For the copper measurement in Example 4-1, the probability density function f(x) = 5.

The cumulative distribution function of the RV X consists of the three expressions:

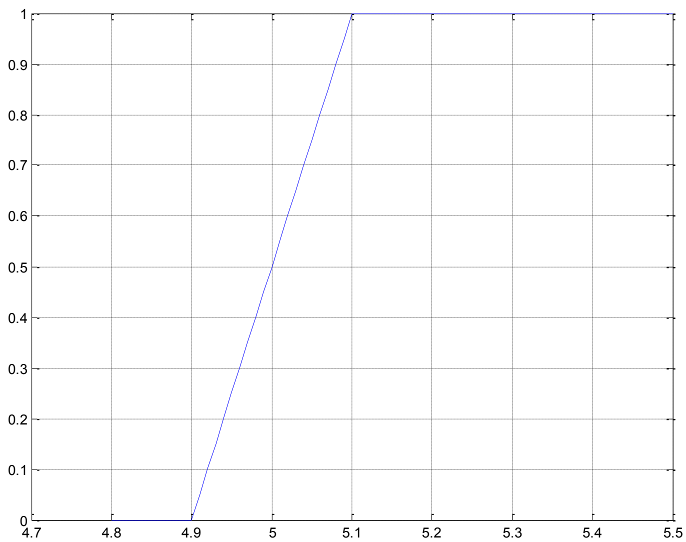
- f(x) = 0, x < 4.9. Therefore, F(x) = 0, for x < 4.9
- f(u) = 5, for 4.9 ≤ u ≤ x, So, F(x) is

  $F(x) = \int_{4.9}^{x} f(u)du = \int_{4.9}^{x} 5du = 5u]_{4.9}^{x} = 5x - 24.5, for$ 4.9 ≤ x ≤ 5.1
- for 5.1 ≤ x, F(x) is
  $F(x) = \int_{4.9}^{x} f(u)du = 1$

$$F(x) = \begin{cases} 0, & x < 4.9 \\ 5x - 24.5, & 4.9 \leq x < 5.1 \\ 1, & 5.1 \leq x \end{cases}$$

```
% ex4_3_and_4.m
%
clear
x0 = 4.8:0.01: (4.9-0.01);
n0 = length(x0);
fx0 = [zeros(1, n0)];
x1 = 4.9:0.01:5.1;
n1 = length(x1);
fx1 = 5.*x1-24.5;    % F(x) = 5x*24.5
x2 = (5.1+0.01):0.01:5.5;
n2 = length(x2);
fx2 = [ones(1,n2)];
x = [x0 x1 x2];
fx =[fx0 fx1 fx2];
plot(x, fx), grid on
```

MATLAB Plot, Figure 4-6:



Ex. 4-4. Hole Diameter. F(x) consists of two expressions:

- for x < 12.5, F(x) = 0
- for 12.5 ≤ x, F(x) equals

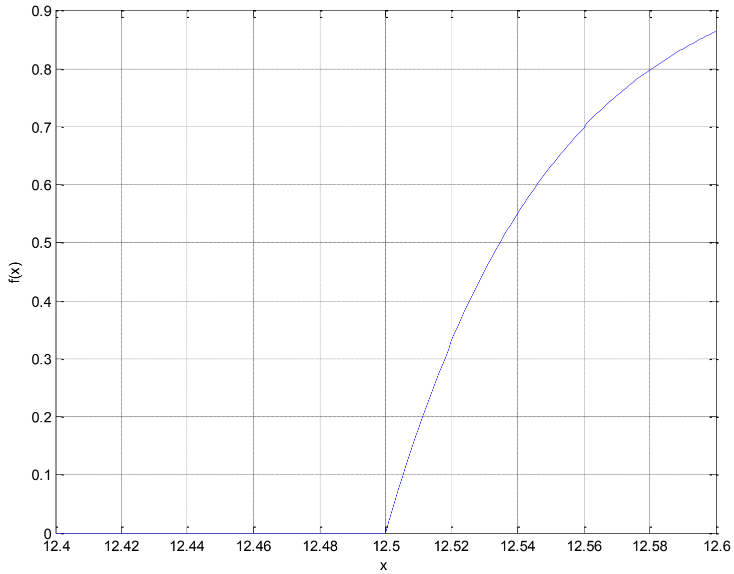
$$F(x) = \int_{12.5}^{x} 20e^{-20(u-12.5)} du = 1 - e^{-20(x-12.5)}$$

Therefore,

$$F(x) = \begin{cases} 0, & x < 12.5 \\ 1 - e^{-20(x-12.5)}, & 12.5 \leq x \end{cases}$$

MATLAB Plot

```
% ex4_4.m
%
clear
x0 = 12.4:0.001: (12.5-0.001);
n0 = length(x0);
fx0 = [zeros(1, n0)];
x1 = (12.5):0.001:12.6;
n1 = length(x1);
fx1 = 1-exp(-20*(x1-12.5));
x = [x0 x1];
fx =[fx0 fx1];
plot(x, fx), grid on
xlabel('x'), ylabel('f(x)');
```



Probability Density Function from the Cumulative Distribution Function, page 113

Given F(x),

$$f(x) = \frac{dF(x)}{dx}$$

As long as the derivative exits.

Ex 4-5. Reaction Time, page 113

The time until a chemical reaction is complete (in milliseconds) is approximated by the cumulative distribution function:

$$F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-0.01x}, & 0 \le x \end{cases}$$

Determine the probability density function of X. What proportion of reaction is complete within 200 ms?

Solution:

From

$$f(x) = \frac{dF(x)}{dx}$$

We can find the probability function f(x):

$$f(x) = \begin{cases} 0, & x < 0 \\ 0.01e^{-0.01x}, & 0 \le x \end{cases}$$

What proportion of reaction is complete within 200 ms?

$$P(X < 200) = F(200) = 1 - e^{-0.01*200} = 1 - e^{-2} = 1 - \frac{1}{e^2} = 1 - \frac{1}{2.7183^2} = 1 - 0.1353 = 0.8647$$

4-4 Mean and Variance of a Continuous Random Variables

The mean or expected value of X:

$$\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

The variance of X, is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2$$

Example 4-6. Electric Current, page 115

For the copper current measurement in Example 4-1, the mean of X is:

$$\mu = E(X) = \int_{4.9}^{5.1} xf(x)dx = \int_{4.9}^{5.1} x \cdot 5dx = \frac{5x^2}{2}\Big]_{4.9}^{5.1} = 5$$

The variance of X is:

$$V(X) = \int_{4.9}^{5.1} (x-\mu)^2 f(x)dx = \int_{4.9}^{5.1} 5 \cdot (x-5)^2 dx = \frac{5(x-5)^3}{3}\Big]_{4.9}^{5.1} = 0.0033$$

The expected value of a function h(X) of a continuous RV is also defined as, page 115

$$E[h(X)] = \int_{-\infty}^{\infty} h(x)f(x)dx$$

Example 4-7, page 115

- In Example 4-1, X is the current measured in ms.
- What is the expected value of power when the resistance is 100 ohms? Use the result $P = I^2*R = 10^{-6}*I^2*R$, where I is the current in mA and R is the resistance in ohms.

Solution:

Now, $h(X) = 10^{-6}*100*X^2$, and the E[h(X)] is

$$E[h(X)] = \int_{4.9}^{5.1} h(x)f(x)dx = \int_{4.9}^{5.1} 10^{-6} \cdot 100 \cdot (x)^2 dx = 0.0001 \frac{x^3}{3}\Big]_{4.9}^{5.1} = 0.0005\, watts = 0.5\, mW$$

Integration by parts, The Fundamental of Integral Calculus, Reference: Calculus and Analytics Geometry, 6th Ed, by Thomas/Finney, pages 451-452

$$\frac{d}{dx}(u \cdot v) = u\frac{dv}{dx} + v\frac{du}{dx}$$

whose differential form is

$$d(uv) = udv + vdu$$

Or

$$u\, dv = d(u\, v) - vdu$$

When this is integrated:

$$\int udv = uv - \int vdu$$

Example 4-8. Hole Diameter, page 115.

For the drilling operation in Example 4-2, the probability density function is

$$f(x) = 20e^{-20(x-12.5)}, x \ \geq 12.5$$

The mean of x is:

$$\mu = E(X) = \int_{12.5}^{\infty} xf(x)dx = \int_{12.5}^{\infty} x \cdot 20e^{-20(x-12.5)}dx$$

Integration by parts can be used to show that

$$E(X) = (-xe^{-20(x-12.5)} - \frac{e^{-20(x-12.5)}}{20})\Big]_{12.5}^{\infty} = 12.5 + 0.05 = 12.55$$

The variance

$$V(X) = \int_{12.5}^{\infty} (x-\mu)^2 f(x)dx = \int_{12.5}^{\infty} (x-12.55)^2 f(x)dx = 0.0025$$

4-5 Continuous Uniform Distribution

A continuous RV X with probability density function

$$f(x) = \frac{1}{b-a}, a \ \leq x \leq b$$

is a continuous uniform RV.

The **mean of continuous RV X** is

$$E(X) = \int_{a}^{b} xf(x)dx = \int_{a}^{b} \frac{x}{b-a}dx = \frac{0.5x^2}{b-a}\Big]_{a}^{b} = \frac{a+b}{2}$$

The **variance of X** is

$$V(X) = \int_a^b (x-\mu)^2 f(x)dx = \int_a^b \frac{(x-\frac{a+b}{2})^2}{b-a}dx = \frac{(x-\frac{a+b}{2})^3}{3(b-a)}\Bigg]_a^b = \frac{(b-a)^2}{12}$$

Example 4-9. Uniform Current, page 117

- In Example 4-1, the RV X has a continuous uniform distribution on [4.9, 5.1].
- The probability density function f(x) = 5, for 4.9 ≤ x≤ 5.1.
- What is the probability that a measurement of current is between 4.95 and 5.0 mA?

$$P(4.95 \le x \le 5.0) = \int_{4.95}^{5} f(x)dx = \int_{4.95}^{5} 5dx = 5(0.05) = 0.25$$

The **mean** can be found with a = 4.9, b = 5.1:

$$E(X) = \frac{a+b}{2} = \frac{5.1-4.9}{2} = \frac{0.2}{2} = 0.1\ mA$$

The **variance**

$$V(X) = \frac{(b-a)^2}{12} = \frac{(5.1-4.9)^2}{12} = \frac{0.04}{12} = 0.0033\ mA^2$$

The **standard deviation**, σ = 0.00577 mA.

4-6 Normal Distribution

- The most widely used model
- De Moivre presented this fundamental result, known as the Central Limit Theorem, in 1733.
- Gauss independently developed a normal distribution nearly 100 years later – Gaussian Distribution

A RV X with probability density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

is a normal RV with parameter μ where $-\infty < \mu < \infty, and\ \sigma > 0$.

Also,

$$E(X) = \mu\ \ and\ V(X) = \sigma^2$$

And the notation $N(\mu, \sigma^2)$ is used to denote the distribution.

The MATLAB Normal Distribution Function (Statistics and Machine Learning Toolbox),
http://www.mathworks.com/help/stats/normal-distribution.html
Example: Compute and plot the Normal pdf, with parameter μ = 0, and σ = 1.

MATLAB Code for Gaussian Distribution, source http://www.matrixlab-examples.com/gaussian-distribution.html

Example 1:

```
% f_gaussian_dist.m
%
function f = f_gaussian_dist(x, mu, sigma)
% x - data to be plotted
% mu - mean
% sigma - standard deviation
p1 = -0.5*((x-mu)/sigma).^2;
p2 = (sigma*sqrt(2*pi));
f = exp(p1) ./ p2;
```

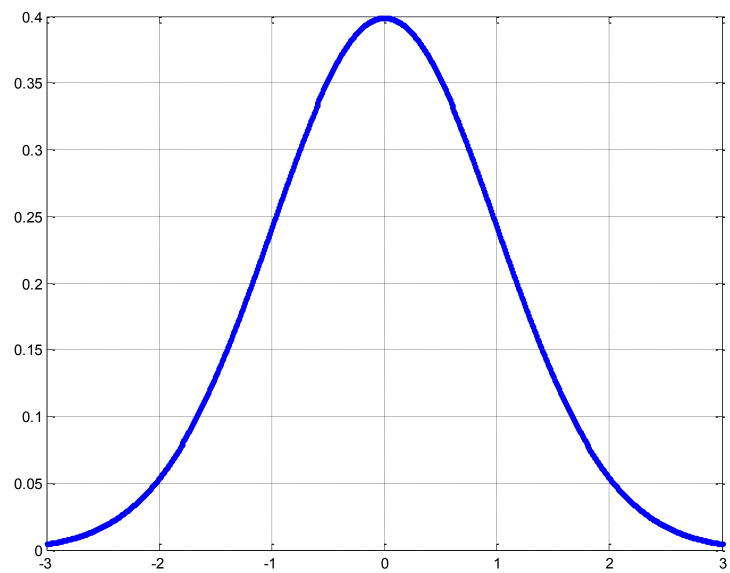
% Call Gaussian Distribution Function and Passed Data

x = [-3: 0.01: 3];

m = 0, s = 1;

f = gaussian_dist(x, m, s);

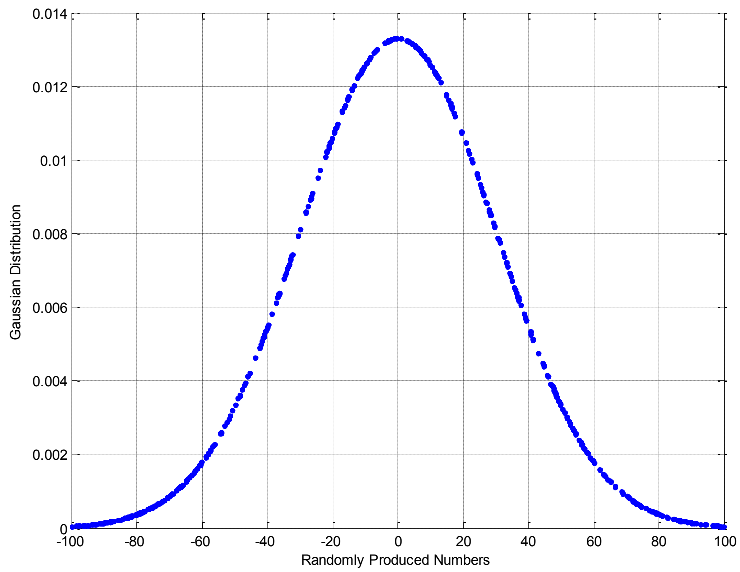
```
plot(x, f, '.'), grid on;
```



Example 2:

```
% plot_gaussian.m
%
% x - data to be plotted
% mu - mean
% sigma - standard deviation
% Call gaussian_dist(x, mu, sigma)
a = -100, b = 100;
x = a + (b-a)* rand(1, 500);
m = (a + b)/2;
s = 30
```

```
f = gaussian_dist(x, m, s);
plot(x, f, '.'), grid on;
xlabel('Randomly Produced Numbers'), ylabel('Gaussian Distribution');
```



Properties associated with a Normal Distribution:

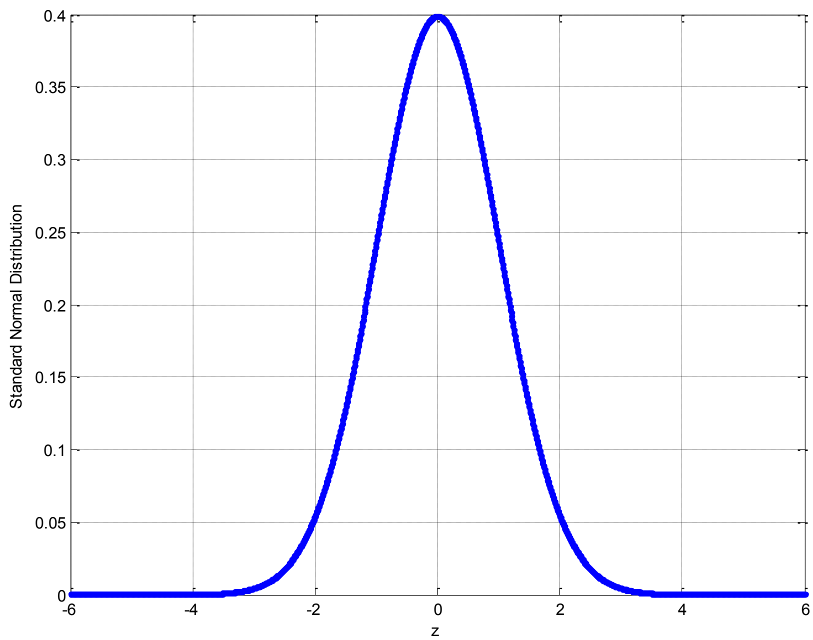
- $p(\mu - \sigma < X < \mu + \sigma) = 0.6827$
- $p(\mu - 2\sigma < X < \mu + 2\sigma) = 0.9549$
- $p(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9977$
- 6σ is often referred to as the width of a normal distribution

Standard Normal Random Variable, page 120

A normal RV with $\mu = 0\ and\ \sigma^2 = 1$ is called a standard random variable and is denoted as Z. The cumulative distribution function of a standard normal random variable is denoted as $\emptyset(z) = P(Z \leq z)$.

MATLAB Script for Standard RV Plot

```
mean = 0; sigma = 1;
z = -6: 0.01: 6;
% Call Gaussian Distribution Function
f = f_gaussian_dist(z, mean, sigma);
plot(z, f, '.'), grid on;
xlabel('z'), ylabel('Standard Normal Distribution');
```

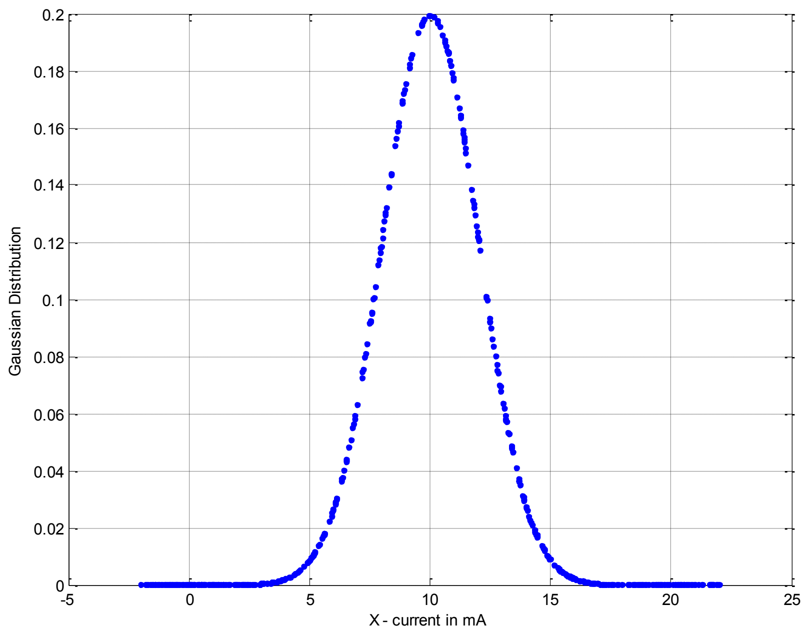


Example 4-10, page 120

- The current measurement in a strip of wire follow a normal distribution with μ = 10 mA, and a variance 0f 4 mA^2 (σ = 2 mA).
- What is the probability that a measurement exceeds 13 mA? Or P(X > 13)

```
% ex4_10.m
%
% x - data to be plotted
% mu - mean
% sigma - standard deviation
% Call gaussian_dist(x, mu, sigma)
m = 10;    % mean 10 mA
s = 2;     % variance 4 mA^2
a = m - 6*s, b = m + 6*s;
x = a + (b-a)* rand(1, 500);

% Call Gaussian Distribution Function
f = f_gaussian_dist(x, m, s);
plot(x, f, '.'), grid on;
xlabel('X - current in mA'), ylabel('Gaussian Distribution');
```



Example 4-11 Standard Normal Distribution, Page 131; also use Table III Cumulative Standard Normal Distribution (pages 742-743).

- Assume that Z is a standard normal RV. Appendix Table III provides probability of the ϕ(z) = P(Z ≤ z).
- The use of Table III to find P(Z ≤ 1.5) = ϕ(1.5): 0.93319;
- P(Z ≤ 1.53) = ϕ(1.53): 0.93699

z	0.00	0.01	0.02	0.03
0	0.5000	0.50399	0.50398	
…	…			
…	…			
1.5	**0.93319**	0.93448	0.93574	**0.93699**

Example 4-12. Calculations for Fig. 4-14.

(1) P(Z > 1.26) = 1 – P(Z ≤ 1.26) = 1-0.89616 = 0.10384
(2) P(Z < -0.86) = 0.19490
(3) P(Z > -1.37) = P(Z < 1.37) = 0.91465
(4) P(-12.5 < Z < 0.37)
  - Find P(Z < 0.37) – P(Z < -1.25) = 0.64431 – 0.10565 = 0.53866
(5) P(Z ≤ -4.6) cannot be found exactly from Table III.
  - P(Z ≤ -3.99) = 0.00003
  - Because P(Z ≤ 4.6) < P(Z ≤ 3.99), so P(Z ≤ 4.6)) is nearly zero

Standardizing a Normal Random Variable

If X is normal RV with E(X) = μ and V(X) = σ^2, the RV

$$Z = \frac{X-\mu}{\sigma}$$

is a normal random variable with E(Z) = 0 and V(Z) = 1. That is, Z is a standard normal variable.

Standardizing to Calculate a Probability

Suppose that X is a normal RV with mean = μ and variance σ^2 . The,

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P(Z \leq z)$$

Where Z is a standard normal RV, and

$$z = \frac{(x-\mu)}{\sigma}$$

is the z-value obtained by standardizing X. The probability is obtained by using Appendix Table III with

$$z = \frac{(x-\mu)}{\sigma}$$

Example 4-13 Normally Distributed Current, page 123

- Suppose that the current measurements is a strip of wire are assumed to follow a normal distribution with a mean of 10 mA, and a variance of 4 mA^2.
- What is the probability that a measurement exceeds 13 mA?

μ = 10, σ = 2; $z = \frac{(x-\mu)}{\sigma}$

Setup Z = (X-10)/2

The relationship between RV X and RV Z is that

- X > 13 corresponds to Z > 1.5

P(X > 13) = P(Z > 1.5) = 1 – P(Z ≤ 1.5) = 1-0.93319 = 0.006681.

Example 4-14 Normally Distributed Current. Continuing Example 4-13, page 124

Q1: What is the probability that a current measurement is between 9 and 11 mA?

P(9 < X 11) = P((9-10/2) < (X-10)/2) < (11-10)/2) = P(-0.5 < Z < 0.5) = P(Z < 0.5) – P(Z < -0.5)
= 0.69146 – 0.30854 = 0.38292

Q2: Determine the value for which the probability that a current measurement is less than this value is 0.98.
P(X < x) = 0.98
P(X < x) = P((X-10)/2 < (x-10)/2) = P(Z(< (x-10)/2) = 0.98

Use Appendix Table III to find the z-value such that P(Z < z) = 0.98, P(Z < 2.06) = 0.97982
Therefore, (x -10)/2 = 2.05, and the standardizing transformation is used in reverse to find x.
x = 2*(2.05) + 10 = 14.1 mA.

Example 4-15 Signal Detection
Assume that in the detection of a digital signal, the background noise follows a normal distribution with a mean of 0 and standard deviation of 0.45 volt. The system assumes a digital 1 has been transmitted when the voltage exceeds 0.9 volt.

Q1: What is the probability of detecting a digital 1 when none was send? => the probability of a false signal.

Solution:
Let RV N denote the voltage of noise. The requested probability is

P(N > 0.9) = P(N/0.45 > 0.9/0.45) = P(Z > 2) = 1- 0.97725 = 0.02275.

Q2: Determine symmetric bounds about 0 that include 99% of all noise readings.
P (-x < N < x) = 0.99
P (-x < N < x) = P(-x/0.45 < N/0.45 < x/0.45) = P(-x/0.45 <Z < x/0.45) = 0.99

From Table III
P(-2.58 < Z < 2.58) = 0.99
Therefore x/0.45 = 2.58, and x = 2.58*(0.45) = 1.16.

Q3: Suppose that when a digital signal is transmitted, then mean of the noise distribution shifts to 1.8 volts. What is the probability that a signal 1 is not detected?

Let the RV S denote the voltage when a digital 1 is transmitted. Then,

P(S < 0.9) = $P = (S < 0.9) = P\left(\frac{S-1.8}{0.45} < \frac{0.9-1.8}{0.45}\right) = P(Z < -2) = 0.02275$

This is the probability of a missed signal.

Example 4-16 Shaft Diameter
The diameter of a shaft in an optical storage drive is normally distributed with mean 0.2508 inch and standard deviation 0.0005 inch. The specification on the shaft are 0.2500±0.0015.
What proportion of the shafts conforms to specifications?

Let X denote the shaft diameter in inches. The requested probability is

0.2485 = 0.2500-0.0015
0.2515 = 0.2500+0.0015

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2508}{0.0005} < Z < \frac{0.2515 - 0.2508}{0.0005}\right) = P(-4.6 < Z < 1.4)$$
$$= P(Z < 1.4) - P(Z < -4.6) = 0.91924 - 0.0000 = 0.91924$$

Most of the nonconforming shafts are too large because the process mean is located very near the upper specification limit. If the process is centered so that the process is equal to the target value of 0.2500.

$$P(0.2485 < X < 0.2515) = P\left(\frac{0.2485 - 0.2500}{0.0005} < Z < \frac{0.2515 - 0.2500}{0.0005}\right) = P(-3 < Z < 3)$$
$$= P(Z < 3) - P(Z < -3) = 0.99865 - 0.00135 = 0.9973$$

By re-centering the process, the yield is increased to approximately 99.73%.