Topics of Discussion

- First-Order System
- Laplace Transform
- Second-Order System
- System Modeling and Analogies
First-Order System – The Spring System

F = m*a = m*dVelocity/dt

Velocity = 1/m \int F dt + Velocity(t0)

Disp = \int Velocity + Disp(t0)

First-Order System – Float-Controlled Tank

\[ \frac{dh}{dt} = K(H - h) \]

\[ h(t) = H(1 - e^{-Kt}) \]
First-Order Differential Equation – RC Circuit

- RC Circuit with a DC Voltage Source
- At \( t = 0 \), the voltage source \( E \) is applied to the circuit

\[
\begin{align*}
V_c &= C \frac{d}{dt} i(t) \\
E &= V_c + V_r \\
E &= \int_0^t i dt \\
E &= \frac{C}{R} t^2 + iR \text{ for } t > 0
\end{align*}
\]

To remove the integral term, we differentiate all terms with respect to \( t \).

\[
\frac{d}{dt} \left( \frac{i}{C} + R \frac{di}{dt} \right) = \frac{dE}{dt}
\]

where \( \frac{dE}{dt} = 0 \) ; \( E = DC \text{ fixed voltage} \)

First-Order Differential Equation (cont.)

- The transient solution is

\[
i(t) = A e^{-t/RC} \text{ for } t > 0
\]

- The constant \( A \) is to be determined from the initial conditions. And the solution \( i(t) \) is where time constant is \( \tau = RC \).

\[
i(t) = \left( \frac{E}{R} \right) e^{-t/(RC)}
\]

- The voltage across capacitor is

\[
v_c(t) = E \left( 1 - e^{-t/RC} \right)
\]

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First-Order Differential Equation (cont.)

- MATLAB Solution: $E = 10V$, $R = 10k$, $C = 0.1\mu F$

- Charging
  - $\tau = R \times C = 1$ millisecond, $V_c(\tau) = E \times 0.636 = 6.36$ volt

  %RC_Transient.m
  E = 10, R = 10E3, C = 0.1E-6;
  tau = R*C; dt = tau/1000;
  t = 0: dt:5*tau;
  Vc = E*(1 - exp(-t./tau));
  plot(t, Vc), grid on
  xlabel('sec'), ylabel('Volt')

- Discharging
  - $\tau = R \times C = 1$ millisecond, $V_c(\tau) = E \times 0.363 = 3.63$ volt

  %RC_Transient.m
  E = 10, R = 10E3, C = 0.1E-6;
  tau = R*C; dt = tau/1000;
  t = 0: dt:5*tau;
  Vc_d = E* exp(-t./tau);
  plot(t, Vc_d), grid on
  xlabel('sec'), ylabel('Volt')
Laplace Transform

- Laplace Transform
  \[ \text{Laplace}[f(t)] = F(s) = \int_0^\infty f(t) e^{-st} \, dt \]
  \[ \text{Laplace}^{-1}[F(s)] = f(t) \]

- Example: Find the Laplace transform for the 5v dc voltage which switch on at time = 0 sec.

\[
F(s) = \text{Laplace}[u(t)] = \text{Laplace}[5] = \int_0^\infty 5 \cdot e^{-st} \, dt = 5 \frac{e^{-st}}{-s}\bigg|_0^\infty
\]
\[
F(s) = \frac{5}{s} [e^{-st} - e^{-0s}] = \frac{5}{s} [0 - 1] = \frac{5}{s}
\]

Transfer Function

- The transfer function of the RC circuit is defined as \( \text{Vout/Vin} \), or
  \[ G(s) = \frac{V_2(s)}{V_1(s)} \]

\[ Z(s) = R + Z_c(s) = R + 1/sC = 2 + 4/s \]
\[ I_e(s) = I_c(s) = \frac{V_1(s)}{R + 1/sC} = \frac{V_1(s)}{2 + 4/s} \]
\[ V_2(s) = I_c(s)Z_c(s) = \frac{V_1(s) \cdot 4}{2 + 4/s} = \frac{V_1(s) \cdot 4}{2s + 4} = \frac{V_1(s) \cdot 2}{s + 2} \]
\[ G(s) = \frac{V_2(s)}{V_1(s)} = \frac{2}{s + 2} \]
Permanent Magnet DC Motor System

- **PM Motor** – constant flux field
- \( I_a \) = Armature current
- \( E_a \) = Armature Emf (Speed voltage)
- \( R_a \) = Armature resistance
- \( V_a \) = Armature terminal voltage
- \( T_{\text{mech}} \) = Mechanical torque
- \( K_m \) = Torque constant

\[ V_a = E_a + I_a R_a \]
\[ I_a = \frac{V_a - E_a}{R_a} \]
\[ E_a = K_a \phi \omega_m = K_m \omega_m \]
\[ T_{\text{mech}} = \frac{E_a I_a}{\omega_m} = K_m I_a \]

Permanent Magnet DC Motor System (Cont.)

- **Motor Data** – Buehler Motor Group
  - \( R = 65\Omega, 80\Omega, 60\Omega, 130\Omega \) average resistance = 83.75\( \Omega \)
  - \( L = 1\text{mH}, 1\text{mH}, 1\text{mH}, 1\text{mH} \) average inductance = 1mH

- 12V @ 160mA (no load)
- 12V @ 520mA (stall current)

\[ \frac{V}{I_a} = 12/52 = R_a \approx 23.1\Omega \]
\[ R_s = R_a \times 0.01 = 0.231\Omega \]
\[ V_a - I_a R_a = 12 - 0.16 \times 23.1 \approx 8.3 \text{ Vdc} = E_a @ 12\text{Vdc} \]
Permanent Magnet DC Motor System (Cont.)

```matlab
%motorcontrol.m
I0 = 0.15; Rd = 1; Re = 1;
Rm = 75;
Lm = 0.1; E = 12;
Im = E/(Rm + Re);
Ron = Re + Rm;
tauon = Lm/Ron;
Roff = Rd + Rm;
tauoff = Lm/Roff;
T = 5*tauoff;
dt = tauon/100;
t = 0:dt:T;
ion = Im*(1-exp(-t/tauon));
ioff = Im*exp(-t/tauoff);
Pr = (ioff.^2)*Ron;
figure(1), plot(t, ioff,'b', t, ion,'r'),
grid on, title('Motor current')
xlabel('time - second'), ylabel('Im Amp')
text(1.7E-3, 0.11,'ion = Im*(1-exp(-t/tauon))')
text(1.7E-3,0.05,'i = Im*exp(-t/tauoff)')
```

Motor current

![Motor current graph]