MET 487 Instrumentation and Automatic Control

System Response
(2 of 2)

Paul I-Hai Lin, Professor
Electrical and Computer Engineering Technology
Purdue University Fort Wayne Campus


Topics of Discussion

- First-Order System
- Laplace Transform
- Second-Order System
- System Modeling and Analogies
First-Order System – The Spring System

- **F = m*a = m*dVelocity/dt**
- **Velocity = 1/m ∫Fdt + Velocity(t0)**
- **Disp = ∫Velocity + Disp(t0)**

First-Order System – Float-Controlled Tank

- 

\[
dh/dt = K(H - h) \\
h(t) = H(1-e^{-Kt})
\]
First-Order Differential Equation – RC Circuit

- RC Circuit with a DC Voltage Source
- At \( t = 0 \), the voltage source \( E \) is applied to the circuit

\[
\begin{align*}
V_c &= \frac{Q}{C} = \frac{1}{C} \int i \cdot dt \\
E &= V_c + V_R \\
E &= \frac{q}{C} + V_R \\
E &= \frac{1}{C} \int i dt + iR \quad \text{for } t > 0
\end{align*}
\]

To remove the integral term, we differentiate all terms with respect to \( t \).

\[
\frac{i}{C} + R \frac{di}{dt} = \frac{dE}{dt}
\]

where \( \frac{dE}{dt} = 0 \), \( E \) = DC fixed voltage

First-Order Differential Equation (cont.)

- The transient solution is

\[
i(t) = Ae^{-t/RC} \quad \text{for } t > 0
\]

- The constant \( A \) is to be determined from the initial conditions. And the solution \( i(t) \) is where time constant is \( \tau = RC \).

\[
i(t) = \left( \frac{E}{R} \right) e^{-t/RC}
\]

- The voltage across capacitor is

\[
v_c(t) = E \left( 1 - e^{-t/RC} \right)
\]
First-Order Differential Equation (cont.)

- MATLAB Solution: $E = 10V$, $R = 10k$, $C = 0.1\mu F$
- Charging
  - $\tau = R \times C = 1$ millisecond, $V_c(\tau) = E \times 0.636 = 6.36$ volt

%RC_Transient.m

```matlab
E = 10, R = 10E3, C = 0.1E-6;
T = R*C; dt = T/1000;
t = 0: dt:5*T;
Vc = E*(1 - exp(-t./T));
plot(t, Vc), grid on
xlabel('sec'), ylabel('Volt')
```

First-Order Differential Equation (cont.)

- MATLAB Solution: $E = 10V$, $R = 10k$, $C = 0.1\mu F$
- Discharging
  - $\tau = R \times C = 1$ millisecond, $V_c(\tau) = E \times 0.363 = 3.63$ volt

%RC_Transient.m

```matlab
E = 10, R = 10E3, C = 0.1E-6;
T = R*C; dt = T/1000;
t = 0: dt:5*T;
Vc_d = E* exp(-t./T);
plot(t, Vc_d), grid on
xlabel('sec'), ylabel('Volt')
```
Laplace Transform

- Laplace Transform
  \[ \text{Laplace} \{ f(t) \} = F(s) = \int_{0}^{\infty} f(t)e^{-st} \, dt \]
  \[ \text{Laplace}^{-1} \{ F(s) \} = f(t) \]

- Example: Find the Laplace transform for the 5v dc voltage which switch on at time = 0 sec.
  \[ F(s) = \text{Laplace} \{ u(t) \} = \text{Laplace} \{ 5 \} = \int_{0}^{\infty} 5 \cdot e^{-st} \, dt = 5 \frac{e^{-st}}{-s} \bigg|_{0}^{\infty} \]
  \[ F(s) = \frac{5}{s} \left[ e^{-st} - e^{-s\cdot0} \right] = \frac{5}{s} [0 - 1] = \frac{5}{s} \]

Transfer Function

- The transfer function of the RC circuit is defined as \( V_{out}/V_{in} \), or \( G(s) = \frac{V_{2}(s)}{V_{1}(s)} \)

\[ Z(s) = R + Z_{c}(s) = R + 1/sC = 2 + 4/s \]
\[ I_{d}(s) = I_{c}(s) = \frac{V_{1}(s)}{R + 1/sC} = \frac{V_{1}(s)}{2 + 4/s} \]
\[ V_{2}(s) = I_{c}(s)Z_{c}(s) = \frac{V_{1}(s) \cdot 4}{2s + 4} = \frac{V_{1}(s) \cdot 2}{s + 2} \]
\[ G(s) = \frac{V_{2}(s)}{V_{1}(s)} = \frac{2}{s + 2} \]
Permanent Magnet DC Motor System

- PM Motor – constant flux field
- Ia = Armature current
- Ea = Armature Emf (Speed voltage)
- Ra = Armature resistance
- Va = Armature terminal voltage
- Tmech = Mechanical torque
- Km = Torque constant

\[
V_a = E_a + I_a R_a \\
I_a = \frac{V_a - E_a}{R_a} \\
E_a = K_a \phi \omega_m = K_m \omega_m \\
T_{mech} = \frac{E_a I_a}{\omega_m} = K_a I_a
\]

Permanent Magnet DC Motor System (Cont.)

- Motor Data – Buehler Motor Group
- R = 65Ω, 80Ω, 60Ω, 130Ω average resistance = 83.75Ω

- L = 1mH, 1mH, 1mH, 1mH average inductance = 1mH

- 12V @ 160mA (no load)
- 12V @ 520mA (stall current)

- V/Ia = 12/.52 = Ra ≈ 23.1Ω
- Rs = Ra * .01 = .231Ω
- Va - IaRa = 12 - .16*23.1 ≈ 8.3 Vdc = Ea @ 12Vdc
Permanent Magnet DC Motor System (Cont.)

```
%motorcontrol.m
I0 = 0.15; Rd = 1; Re = 1;
Rm = 75;
Lm = 0.1; E = 12;
Im = E/(Rm + Re);
Ron = Re + Rm;
tauon = Lm/Ron;
Roff = Rd + Rm;
tauoff = Lm/Roff;
T = 5*tauoff;
dt = tauon/100;
t = 0:dt:T;
ion = Im*(1-exp(-t/tauon));
ioff = Im*exp(-t/tauoff);
Pr = (ioff.^2)*Ron;
figure(1), plot(t, ioff,'b', t, ion,'r'),
grid on, title('Motor current')
xlabel('time - second'), ylabel('Im Amp')
text(1.7E-3, 0.11,'ion = Im*(1-exp(-t/tauon))')
text(1.7E-3,0.05,'i = Im*exp(-t/tauoff)')
```