### Ideal Gas

\[ \bar{P} \bar{v} = RT \]

\[ R = c_p - c_v \quad k = c_p / c_v \]

**Cycle**

\[ \oint \delta Q = \oint \delta W \]

### Closed System–Control Mass

\[ 1Q_2 = U_2 - U_1 + \frac{m(V_2^2 - V_1^2)}{2} + mg(Z_2 - Z_1) + 1W_2 \]

\[ m(s_2 - s_1) \geq \int_1^2 \frac{\delta Q}{T} = \int_1^2 \frac{\delta Q}{T} + 1S_{gen} \]

### Open System–Control Volume

\[ \sum_i \dot{m}_i - \sum_e \dot{m}_e = \frac{dmcv}{dt} \quad \dot{m} = \dot{V} / v = VA/v \]

\[ \dot{Q}_{cv} - W_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right) = \frac{dE_{cv}}{dt} \]

\[ \frac{dS_{cv}}{dt} \geq \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} = \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum \frac{\dot{Q}_{cv}}{T} + S_{gen} \]

### Gibbs Relations

\[ Tds = du + Pdv = dh - vdp \]

**Internal Energy, Enthalpy, and Entropy Change–Ideal Gas**

\[ u_2 - u_1 = \int_1^2 c_v \, dT, \quad h_2 - h_1 = \int_1^2 c_p \, dT, \]

\[ s_2 - s_1 = \int_1^2 \frac{c_v}{T} \, dT - R \ln \frac{P_2}{P_1} = \int_1^2 \frac{c_v}{T} \, dT + R \ln \frac{v_2}{v_1} = s_{T_2}^0 - s_{T_1}^0 - R \ln \frac{P_2}{P_1} \]

**Isentropic Process–Ideal Gas** with constant specific heats

\[ \frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left( \frac{v_1}{v_2} \right)^k \]

### Shaft Work

**– reversible, neglect KE & PE**

\[ \dot{w} = - \int_i^e v \, dp \]

**– reversible, polytropic**

\[ \dot{w} = - \frac{n}{n-1} (P_e v_e - P_i v_i) \]
Closed System—Exergy and Irreversibility

Combined first and second law

\[ 1W_2 = U_1 - U_2 - T_0(S_1 - S_2) + \sum_j Q_j \left( 1 - \frac{T_0}{T_j} \right) - T_0S_{2}^{\text{gen}} \]

Useful work

\[ 1W_2^u = 1W_2 - P_0(V_2 - V_1) = U_1 - U_2 - T_0(S_1 - S_2) + P_0(V_1 - V_2) + \sum_j Q_j \left( 1 - \frac{T_0}{T_j} \right) - T_0S_{2}^{\text{gen}} \]

Exergy or nonflow availability per unit mass

\[ \phi = (u - u_0) - T_0(s - s_0) + P_0(v - v_0) \]

Useful work

\[ 1W_2^u = m(\phi_1 - \phi_2) + \sum_j Q_j \left( 1 - \frac{T_0}{T_j} \right) - T_0S_{2}^{\text{gen}} \]

Irreversibility

\[ 1I_2 = 1W_2^{\text{rev}} - W_2 = 1W_2^{\text{max}} - 1W_2^u = T_0S_{2}^{\text{gen}} = T_0 \left[ (S_2 - S_1) - \sum_j \frac{Q_j}{T_j} \right] \]

Exergy balance

\[ \Phi_2 - \Phi_1 = m(\phi_2 - \phi_1) = -[1W_2 - P_0(V_2 - V_1)] + \sum_j Q_j \left( 1 - \frac{T_0}{T_j} \right) - T_0S_{2}^{\text{gen}} \]

Control Volume—Exergy and Irreversibility

Combined first and second law

\[ \dot{W}_{cv} = \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i - T_0s_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e - T_0s_e \right) + \sum_j \dot{Q}_j \left( 1 - \frac{T_0}{T_j} \right) - T_0\dot{S}_{\text{gen}} \]

Flow availability or exergy per unit mass

\[ \psi = (h + V^2/2 + gz - T_0s) - (h_0 + gz_0 - T_0s_0) \]

Reversible work

\[ \dot{W}_{cv}^{\text{rev}} = \sum_i \dot{m}_i \psi_i - \sum_e \dot{m}_e \psi_e + \sum_j \dot{Q}_j \left( 1 - \frac{T_0}{T_j} \right) \]

Irreversibility

\[ \dot{I}_{cv} = \dot{W}_{cv}^{\text{rev}} - \dot{W}_{cv} = T_0\dot{S}_{\text{gen}} = T_0 \left[ \sum_e \dot{m}_e s_e - \sum_i \dot{m}_i s_i - \sum_j \frac{\dot{Q}_j}{T} \right] \]

Exergy balance

\[ \dot{\Psi}_2 - \dot{\Psi}_1 = \sum_e \dot{m}_e \psi_e - \sum_i \dot{m}_i \psi_i = -\dot{W}_{cv} + \sum_j \dot{Q}_j \left( 1 - \frac{T_0}{T_j} \right) - T_0\dot{S}_{\text{gen}} \]
Gas Mixtures

Composition

- Gravametric analysis (mass basis)
  \[ m = m_1 + m_2 + m_3 + \cdots = \sum_i m_i \]
  \[ 1 = \frac{m_1}{m} + \frac{m_2}{m} + \frac{m_3}{m} + \cdots = \sum_i \frac{m_i}{m} \]
  or with \( c_i \equiv m_i/m \), then \( \sum c_i = 1 \).

- Molar analysis (mole basis)
  \[ n = n_1 + n_2 + n_3 + \cdots = \sum_i n_i \]
  \[ 1 = \frac{n_1}{n} + \frac{n_2}{n} + \frac{n_3}{n} + \cdots = \sum_i \frac{n_i}{n} \]
  or with \( y_i \equiv n_i/n \), then \( \sum y_i = 1 \).

- Molecular weight
  \[ M_i = \frac{m_i}{n_i} \]
  \[ M = \frac{m}{n} = \frac{\sum n_i M_i}{n} = \sum y_i M_i \]

- Conversion
  mole basis → mass basis
  \[ c_i = \frac{m_i}{m} = \frac{n_i M_i}{\sum n_j M_j} = \frac{n_i M_i/n}{\sum n_j M_j/n} = \frac{y_i M_i}{\sum y_j M_j} \]
  mass basis → mole basis
  \[ y_i = \frac{n_i}{n} = \frac{m_i/M_i}{\sum m_j/M_j} = \frac{m_i/(M_i m)}{\sum m_j(M_j m)} = \frac{c_i/M_i}{\sum c_j/M_j} \]

Ideal Gas Relationships

\[ PV = n\bar{R}T = mRT \]

\[ \sum n_i = n \quad \text{and} \quad \sum m_i = m \]

\[ \bar{R} = 8.314 \frac{\text{kJ}}{\text{kmol K}} = 1545 \frac{\text{ft lbf}}{\text{lb mol R}} = 1.986 \frac{\text{Btu}}{\text{lb mol R}} \quad \text{and} \quad R = \frac{\bar{R}}{M} \]
• Dalton Model - components behave as an ideal gas at $T$ and $V$ of mixture

$$P_i = n_i \bar{R}T/V = m_i R_i T/V$$

$$\frac{P_i}{P} = \frac{n_i \bar{R}T/V}{nRT/V} = \frac{n_i}{n} = y_i \implies P_i = y_i P \quad \therefore \quad \sum P_i = \sum y_i P = P \sum y_i = P$$

• Amagat-Leduc Model - components behave as an ideal gas at $T$ and $P$ of mixture

$$V_i = n_i \bar{R}T/P = m_i R_i T/P$$

$$\frac{V_i}{V} = \frac{n_i \bar{R}T/P}{nRT/P} = \frac{n_i}{n} = y_i \implies V_i = y_i V \quad \therefore \quad \sum V_i = \sum y_i V = V \sum y_i = V$$

• Note that $R = \sum c_i R_i$

**Properties**

$$U = \sum U_i \quad \text{where} \quad U_i = n_i \bar{u}_i \quad \text{or} \quad U_i = m_i u_i$$

$$U = n\bar{u} = \sum n_i \bar{u}_i \implies \bar{u} = \sum y_i \bar{u}_i \quad \text{or} \quad U = mu = \sum m_i u_i \implies u = \sum c_i u_i$$

$$\bar{c}_v \equiv \left( \frac{\partial \bar{u}}{\partial T} \right)_v = \sum y_i \left( \frac{\partial \bar{u}_i}{\partial T} \right)_v = \sum y_i \bar{c}_v_i \quad \text{or} \quad c_v \equiv \left( \frac{\partial u_i}{\partial T} \right)_v = \sum c_i \left( \frac{\partial u_i}{\partial T} \right)_v = \sum c_i c_{v_i}$$

$$H = \sum H_i \quad \text{where} \quad H_i = n_i \bar{h}_i \quad \text{or} \quad H_i = m_i h_i$$

$$H = n\bar{h} = \sum n_i \bar{h}_i \implies \bar{h} = \sum y_i \bar{h}_i \quad \text{or} \quad H = mh = \sum m_i h_i \implies h = \sum c_i h_i$$

$$\bar{c}_p \equiv \left( \frac{\partial \bar{h}}{\partial T} \right)_p = \sum y_i \left( \frac{\partial \bar{h}_i}{\partial T} \right)_p = \sum y_i \bar{c}_{p_i} \quad \text{or} \quad c_p \equiv \left( \frac{\partial h_i}{\partial T} \right)_p = \sum c_i \left( \frac{\partial h_i}{\partial T} \right)_p = \sum c_i c_{p_i}$$

$$S = \sum S_i \quad \text{where} \quad S_i = n_i \bar{s}_i \quad \text{or} \quad S_i = m_i s_i$$

$$S = n\bar{s} = \sum n_i \bar{s}_i \implies \bar{s} = \sum y_i \bar{s}_i \quad \text{or} \quad S = ms = \sum m_i s_i \implies s = \sum c_i s_i$$

$$d\bar{s} = \sum y_i \bar{c}_{p_i} \frac{dT}{T} - \sum y_i \bar{R}_i \frac{dP_i}{P_i} \quad \text{or} \quad ds = \sum c_{p_i} \frac{dT}{T} - \sum c_i R_i \frac{dP_i}{P_i}$$

$$M = \frac{\bar{u}}{u} = \frac{\bar{h}}{h} = \frac{\bar{s}}{s} = \frac{\bar{c}_v}{c_v} = \frac{\bar{c}_p}{c_p} \quad \text{and} \quad M_i = \frac{\bar{u}_i}{u_i} = \frac{\bar{h}_i}{h_i} = \frac{\bar{s}_i}{s_i} = \frac{\bar{c}_{v_i}}{c_{v_i}} = \frac{\bar{c}_{p_i}}{c_{p_i}}$$