HEAT TRANSFER

There are three modes of heat transfer: conduction, convection, and radiation.

BASIC HEAT TRANSFER RATE EQUATIONS

Conduction
Fourier’s Law of Conduction
\[ \dot{Q} = -kA \frac{dT}{dx}, \text{where} \]
\[ \dot{Q} = \text{rate of heat transfer (W)} \]
\[ k = \text{the thermal conductivity [W/(m} \cdot \text{K}]} \]
\[ A = \text{the surface area perpendicular to direction of heat transfer (m}^2) \]

Convection
Newton’s Law of Cooling
\[ \dot{Q} = hA(T_w - T_\infty), \text{where} \]
\[ h = \text{the convection heat transfer coefficient of the fluid [W/(m}^2 \cdot \text{K}]} \]
\[ A = \text{the convection surface area (m}^2) \]
\[ T_w = \text{the wall surface temperature (K)} \]
\[ T_\infty = \text{the bulk fluid temperature (K)} \]

Radiation
The radiation emitted by a body is given by
\[ \dot{Q} = \varepsilon \sigma A T^4, \text{where} \]
\[ \varepsilon = \text{the emissivity of the body} \]
\[ \sigma = \text{the Stefan-Boltzmann constant} \]
\[ = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4) \]
\[ A = \text{the body surface area (m}^2) \]
\[ T = \text{the absolute temperature (K)} \]

CONDUCTION

Conduction Through a Plane Wall
\[ \dot{Q} = -kA \frac{T_2 - T_1}{L}, \text{where} \]
\[ A = \text{wall surface area normal to heat flow (m}^2) \]
\[ L = \text{wall thickness (m)} \]
\[ T_1 = \text{temperature of one surface of the wall (K)} \]
\[ T_2 = \text{temperature of the other surface of the wall (K)} \]
To evaluate Surface or Intermediate Temperatures:

\[ \dot{Q} = \frac{T_s - T_2}{R_A} = \frac{T_2 - T_1}{R_B} \]

**Steady Conduction with Internal Energy Generation**

The equation for one-dimensional steady conduction is

\[ \frac{d^2T}{dx^2} + \frac{Q_{gen}}{k} = 0, \]

where

\( Q_{gen} \) = the heat generation rate per unit volume (W/m³)

For a Plane Wall

\[ T(x) = \frac{Q_{gen}L^2}{2k} \left( 1 - \frac{x^2}{L^2} \right) + \left( \frac{T_2 - T_{s1}}{2} \right) \left( \frac{x}{L} \right) + \left( \frac{T_{s1} - T_{s2}}{2} \right) \]

\( \dot{Q}_1' + \dot{Q}_2' = 2Q_{gen}L, \) where

\( \dot{Q}' = \) the rate of heat transfer per area (heat flux) (W/m²)

\[ \dot{Q}_1' = k \left( \frac{dT}{dx} \right)_{-L} \text{ and } \dot{Q}_2' = k \left( \frac{dT}{dx} \right)_L \]

For a Long Circular Cylinder

\[ T(r) = \frac{Q_{gen}r_0^2}{4k} \left( 1 - \frac{r^2}{r_0^2} \right) + T_s \]

\[ \dot{Q}' = \pi r_0^2 \dot{Q}_{gen}, \] where

\( \dot{Q}' = \) the heat transfer rate from the cylinder per unit length of the cylinder (W/m)

**Transient Conduction Using the Lumped Capacitance Method**

The lumped capacitance method is valid if

\[ \frac{hV}{kA_s} \ll 1, \]

where

\( h = \) the convection heat transfer coefficient of the fluid [W/(m²·K)]

\( V = \) the volume of the body (m³)

\( k = \) thermal conductivity of the body [W/(m·K)]

\( A_s = \) the surface area of the body (m²)

**Constant Fluid Temperature**

If the temperature may be considered uniform within the body at any time, the heat transfer rate at the body surface is given by

\[ \dot{Q} = hA_s (T - T_\infty) = -\rho V c_p \left( \frac{dT}{dt} \right), \]

where

\( T = \) the body temperature (K)

\( T_\infty = \) the fluid temperature (K)

\( \rho = \) the density of the body (kg/m³)

\( c_p = \) the heat capacity of the body [J/(kg·K)]

\( t = \) time (s)

The temperature variation of the body with time is

\[ T - T_\infty = (T_i - T_\infty) e^{-\beta t}, \]

where

\[ \beta = \frac{hA_s}{\rho V c_p} \]

and

\( \tau = \) time constant (s)

The total heat transferred (\( Q_{total} \)) up to time \( t \) is

\[ Q_{total} = \rho V c_p (T_i - T), \]

where

\( T_i = \) initial body temperature (K)
Variable Fluid Temperature

If the ambient fluid temperature varies periodically according to the equation

\[ T_\infty = T_{\infty,\text{mean}} + \frac{1}{2} (T_{\infty,\text{max}} - T_{\infty,\text{min}}) \cos(\omega t) \]

The temperature of the body, after initial transients have died away, is

\[ T = \beta \frac{1}{2} (T_{\infty,\text{max}} - T_{\infty,\text{min}}) \cos \left( \omega t - \tan^{-1} \left( \frac{\omega}{\beta} \right) \right) + T_{\infty,\text{mean}} \]

Fins

For a straight fin with uniform cross section (assuming negligible heat transfer from tip),

\[ \dot{Q} = \sqrt{hP}kA_c(T_b - T_\infty) \tanh(mL_c), \]

where

- \( h \) = the convection heat transfer coefficient of the fluid [W/(m²•K)]
- \( P \) = perimeter of exposed fin cross section (m)
- \( k \) = fin thermal conductivity [W/(m•K)]
- \( A_c \) = fin cross-sectional area (m²)
- \( T_b \) = temperature at base of fin (K)
- \( T_\infty \) = fluid temperature (K)
- \( m = \sqrt{\frac{hP}{kA_c}} \)
- \( L_c = L + \frac{A_c}{P}, \) corrected length of fin (m)

Rectangular Fin

\[ P = 2w + 2t \]

\[ A_c = wt \]

Pin Fin

\[ P = \pi D \]

\[ A_c = \frac{\pi D^2}{4} \]

\[ T_\infty, h \]

\[ T_b \]

\[ w \]

\[ L \]

\[ t \]

\[ D \]

CONVECTION

Terms

- \( D \) = diameter (m)
- \( \bar{h} \) = average convection heat transfer coefficient of the fluid [W/(m²•K)]
- \( L \) = length (m)
- \( \overline{Nu} \) = average Nusselt number
- \( Pr \) = Prandtl number = \( \frac{cp\mu}{k} \)
- \( u_m \) = mean velocity of fluid (m/s)
- \( u_\infty \) = free stream velocity of fluid (m/s)
- \( \mu \) = dynamic viscosity of fluid [kg/(s•m)]
- \( \rho \) = density of fluid (kg/m³)

External Flow

In all cases, evaluate fluid properties at average temperature between that of the body and that of the flowing fluid.

Flat Plate of Length \( L \) in Parallel Flow

\[ \Re_L = \frac{\hat{u}_\infty L}{\mu} \]

\[ \overline{Nu}_L = \frac{\bar{h}L}{k} = 0.6640 \Re_L^{1/3} Pr^{1/3} \quad (\Re_L < 10^5) \]

\[ \overline{Nu}_L = \frac{\bar{h}L}{k} = 0.0366 \Re_L^{0.8} Pr^{1/3} \quad (\Re_L > 10^5) \]

Cylinder of Diameter \( D \) in Cross Flow

\[ \Re_D = \frac{\hat{u}_\infty D}{\mu} \]

\[ \overline{Nu}_D = \frac{\bar{h}D}{k} = C \Re_D^n Pr^{1/3}, \]

where

<table>
<thead>
<tr>
<th>( Re_D )</th>
<th>( C )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - 4</td>
<td>0.989</td>
<td>0.330</td>
</tr>
<tr>
<td>4 - 40</td>
<td>0.911</td>
<td>0.385</td>
</tr>
<tr>
<td>40 - 400</td>
<td>0.683</td>
<td>0.466</td>
</tr>
<tr>
<td>4,000 - 40,000</td>
<td>0.193</td>
<td>0.618</td>
</tr>
<tr>
<td>40,000 - 250,000</td>
<td>0.0266</td>
<td>0.805</td>
</tr>
</tbody>
</table>

Flow Over a Sphere of Diameter, \( D \)

\[ \overline{Nu}_D = \frac{\bar{h}D}{k} = 2.0 + 0.60 \Re_D^{1/3} Pr^{1/3}, \]

\( (1 < \Re_D < 70,000; 0.6 < Pr < 400) \)

Internal Flow

\[ \Re_D = \frac{\hat{u}_\infty D}{\mu} \]

Laminar Flow in Circular Tubes

For laminar flow (\( \Re_D < 2300 \)), fully developed conditions

\( Nu_D = 4.36 \) (uniform heat flux)

\( Nu_D = 3.66 \) (constant surface temperature)
For laminar flow (Re_D < 2300), combined entry length with constant surface temperature

\[ Nu_D = 1.86 \left( \frac{Re_D Pr}{D^3} \right)^{1/3} \left( \frac{H_b}{H_s} \right)^{0.14} , \text{ where} \]

\( L = \text{length of tube (m)} \)
\( D = \text{tube diameter (m)} \)
\( \mu_b = \text{dynamic viscosity of fluid [kg/(s\cdot m)] at bulk temperature of fluid, } T_b \)
\( \mu_s = \text{dynamic viscosity of fluid [kg/(s\cdot m)] at inside surface temperature of the tube, } T_s \)

Turbulent Flow in Circular Tubes
For turbulent flow (Re_D > 10^4, Pr > 0.7) for either uniform surface temperature or uniform heat flux condition, Sieder-Tate equation offers good approximation:

\[ Nu_D = 0.027 Re_D^{0.8} Pr^{1/3} \left( \frac{H_b}{H_s} \right)^{0.14} \]

Non-Circular Ducts
In place of the diameter, D, use the equivalent (hydraulic) diameter \((D_H)\) defined as

\[ D_H = \frac{4 \times \text{cross-sectional area}}{\text{wetted perimeter}} \]

Circular Annulus (D_s > D_i)
In place of the diameter, D, use the equivalent (hydraulic) diameter \((D_H)\) defined as

\[ D_H = D_s - D_i \]

Liquid Metals (0.003 < Pr < 0.05)

\[ Nu_D = 6.3 + 0.0167 Re_D^{0.8} Pr^{0.93} \text{ (uniform heat flux)} \]
\[ Nu_D = 7.0 + 0.025 Re_D^{0.8} Pr^{0.8} \text{ (constant wall temperature)} \]

Condensation of a Pure Vapor
On a Vertical Surface

\[ \bar{Nu}_L = \frac{\bar{h}L}{k} = 0.943 \left[ \frac{\rho_l^2 g h_L L^3}{\mu_l k_l (T_{sat} - T_s)} \right]^{0.25} , \text{ where} \]

\( \rho_l = \text{density of liquid phase of fluid (kg/m}^3\rangle \)
\( g = \text{gravitational acceleration (9.81 m/s}^2\rangle \)
\( h_L = \text{latent heat of vaporization [J/kg]} \)
\( L = \text{length of surface [m]} \)
\( \mu_l = \text{dynamic viscosity of liquid phase of fluid [kg/(s\cdot m)]} \)
\( k_l = \text{thermal conductivity of liquid phase of fluid [W/(m\cdot K)]} \)
\( T_{sat} = \text{saturation temperature of fluid [K]} \)
\( T_s = \text{temperature of vertical surface [K]} \)

Note: Evaluate all liquid properties at the average temperature between the saturated temperature, \(T_{sat}\), and the surface temperature, \(T_s\).

Outside Horizontal Tubes

\[ Nu_D = \frac{\bar{h}D}{k} = 0.729 \left[ \frac{\rho_l^2 g h_L D^3}{\mu_l k_l (T_{sat} - T_s)} \right]^{0.25} , \text{ where} \]

\( D = \text{tube outside diameter (m)} \)

Note: Evaluate all liquid properties at the average temperature between the saturated temperature, \(T_{sat}\), and the surface temperature, \(T_s\).

Natural (Free) Convection
Vertical Flat Plate in Large Body of Stationary Fluid
Equation also can apply to vertical cylinder of sufficiently large diameter in large body of stationary fluid.

\[ h = C \left( \frac{k}{L} \right) Ra_L^n , \text{ where} \]

\( L = \text{the length of the plate (cylinder) in the vertical direction} \)
\( Ra_L = \text{Rayleigh Number} = \frac{g \beta (T_s - T_\infty) L^3}{\nu^2 \Pr} \)
\( T_s = \text{surface temperature (K)} \)
\( T_\infty = \text{fluid temperature (K)} \)
\( \beta = \text{coefficient of thermal expansion (1/K)} \)

(For an ideal gas: \( \beta = \frac{2}{T_s + T_\infty} \) with \( T \) in absolute temperature)

\( \nu = \text{kinematic viscosity (m}^2/\text{s)} \)

<table>
<thead>
<tr>
<th>Range of Ra_L</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^4 \text{ - } 10^9)</td>
<td>0.59</td>
<td>1/4</td>
</tr>
<tr>
<td>(10^9 \text{ - } 10^{13})</td>
<td>0.10</td>
<td>1/3</td>
</tr>
</tbody>
</table>

Long Horizontal Cylinder in Large Body of Stationary Fluid

\[ \bar{h} = C \left( \frac{k}{D} \right) Ra_D^n , \text{ where} \]

\[ Ra_D = \frac{g \beta (T_s - T_\infty) D^3}{\nu^2 \Pr} \]

<table>
<thead>
<tr>
<th>Ra_D</th>
<th>C</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^{-3} \text{ - } 10^{-2})</td>
<td>1.02</td>
<td>0.148</td>
</tr>
<tr>
<td>(10^{2} \text{ - } 10^{4})</td>
<td>0.850</td>
<td>0.188</td>
</tr>
<tr>
<td>(10^{4} \text{ - } 10^{7})</td>
<td>0.480</td>
<td>0.250</td>
</tr>
<tr>
<td>(10^{7} \text{ - } 10^{12})</td>
<td>0.125</td>
<td>0.333</td>
</tr>
</tbody>
</table>

Heat Exchangers
The rate of heat transfer in a heat exchanger is

\[ Q = U A F \Delta T_{lm} \text{ where} \]

\( A = \text{any convenient reference area (m}^2\rangle \)
\( F = \text{heat exchanger configuration correction factor} \)
\( (F = 1 \text{ if temperature change of one fluid is negligible)} \)
\( U = \text{overall heat transfer coefficient based on area } A \text{ and the log mean temperature difference [W/(m}^2\cdot\text{K}]} \)
\( \Delta T_{lm} = \text{log mean temperature difference (K)} \)
Heat Exchangers (cont.)

Overall Heat Transfer Coefficient for Concentric Tube and Shell-and-Tube Heat Exchangers

\[
\frac{1}{UA} = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln \left( \frac{D_i}{D_o} \right)}{2 \pi k L} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o},
\]

where

- \( A_i \) = inside area of tubes (m²)
- \( A_o \) = outside area of tubes (m²)
- \( D_i \) = inside diameter of tubes (m)
- \( D_o \) = outside diameter of tubes (m)
- \( h_i \) = convection heat transfer coefficient for inside of tubes [W/(m²•K)]
- \( h_o \) = convection heat transfer coefficient for outside of tubes [W/(m²•K)]
- \( k \) = thermal conductivity of tube material [W/(m•K)]
- \( R_{fi} \) = fouling factor for inside of tube [(m²•K)/W]
- \( R_{fo} \) = fouling factor for outside of tube [(m²•K)/W]

Log Mean Temperature Difference (LMTD)

For counterflow in tubular heat exchangers

\[
\Delta T_{lm} = \frac{(T_{Ho} - T_{Ci}) - (T_{Hi} - T_{Co})}{\ln \left( \frac{T_{Ho} - T_{Ci}}{T_{Hi} - T_{Co}} \right)}
\]

For parallel flow in tubular heat exchangers

\[
\Delta T_{lm} = \frac{(T_{Ho} - T_{Co}) - (T_{Hi} - T_{Ci})}{\ln \left( \frac{T_{Ho} - T_{Co}}{T_{Hi} - T_{Ci}} \right)}
\]

\( \Delta T_{lm} \) = log mean temperature difference (K)

\( T_{Hi} \) = inlet temperature of the hot fluid (K)

\( T_{Ho} \) = outlet temperature of the hot fluid (K)

\( T_{Ci} \) = inlet temperature of the cold fluid (K)

\( T_{Co} \) = outlet temperature of the cold fluid (K)

Heat Exchanger Effectiveness, \( \varepsilon \)

\[
\varepsilon = \frac{Q}{Q_{\text{max}}} = \frac{\text{actual heat transfer rate}}{\text{maximum possible heat transfer rate}}
\]

\[
\varepsilon = \frac{C_H (T_{Hi} - T_{Ho})}{C_{\text{min}} (T_{Hi} - T_{Ci})} \quad \text{or} \quad \varepsilon = \frac{C_C (T_{Co} - T_{Ci})}{C_{\text{min}} (T_{Hi} - T_{Ci})}
\]

where

\( C = m c_p \) = heat capacity rate (W/K)

\( C_{\text{min}} \) = smaller of \( C_C \) or \( C_H \)

Number of Transfer Units (NTU)

\[
NTU = \frac{UA}{C_{\text{min}}}
\]

Effectiveness-NTU Relations

\[
C_r = \frac{C_{\text{min}}}{C_{\text{max}}} = \text{heat capacity ratio}
\]

For parallel flow concentric tube heat exchanger

\[
\varepsilon = \frac{1 - \exp\left[ -NTU(1 + C_r) \right]}{1 + C_r}
\]

\[
NTU = -\log \left( \frac{1 - \varepsilon (1 + C_r)}{C_r} \right)
\]

For counterflow concentric tube heat exchanger

\[
\varepsilon = \frac{1 - \exp\left[ -NTU(1 - C_r) \right]}{1 - C_r \exp\left[ -NTU(1 - C_r) \right]}
\]

\[
NTU = \frac{\frac{\varepsilon}{C_r - 1}}{\frac{1}{C_r} - 1}
\]

RADIATION

Types of Bodies

Any Body

\( \alpha + \rho + \tau = 1 \), where

\( \alpha \) = absorptivity (ratio of energy absorbed to incident energy)

\( \rho \) = reflectivity (ratio of energy reflected to incident energy)

\( \tau \) = transmissivity (ratio of energy transmitted to incident energy)

Opaque Body

For an opaque body: \( \alpha + \rho = 1 \)

Gray Body

A gray body is one for which

\( \alpha = \varepsilon, \quad 0 < \alpha < 1; \quad 0 < \varepsilon < 1 \), where

\( \varepsilon \) = the emissivity of the body

For a gray body: \( \varepsilon + \rho = 1 \)

Real bodies are frequently approximated as gray bodies.

Black Body

A black body is defined as one which absorbs all energy incident upon it. It also emits radiation at the maximum rate for a body of a particular size at a particular temperature. For such a body

\( \alpha = \varepsilon = 1 \)
Shape Factor (View Factor, Configuration Factor) Relations

Reciprocity Relations

\[ A_i F_{ij} = A_j F_{ji}, \]
where
\[ A_i = \text{surface area (m}^2\text{)} \text{ of surface } i \]

\[ F_{ij} = \text{shape factor (view factor, configuration factor); fraction of the radiation leaving surface } i \text{ that is intercepted by surface } j; \ 0 \leq F_{ij} \leq 1 \]

Summation Rule for N Surfaces

\[ \sum_{j=1}^{N} F_{ij} = 1 \]

Net Energy Exchange by Radiation between Two Bodies

Body Small Compared to its Surroundings

\[ \dot{Q}_{12} = \varepsilon A \sigma (T_1^4 - T_2^4), \]
where
\[ \dot{Q}_{12} = \text{the net heat transfer rate from the body (W)} \]
\[ \varepsilon = \text{the emissivity of the body} \]
\[ \sigma = \text{the Stefan-Boltzmann constant} \]
\[ [\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)] \]
\[ A = \text{the body surface area (m}^2\text{)} \]
\[ T_1 = \text{the absolute temperature [K] of the body surface} \]
\[ T_2 = \text{the absolute temperature [K] of the surroundings} \]

Net Energy Exchange by Radiation between Two Black Bodies

The net energy exchange by radiation between two black bodies that see each other is given by

\[ \dot{Q}_{12} = A_1 F_{12} \sigma (T_1^4 - T_2^4) \]

Net Energy Exchange by Radiation between Two Diffuse-Gray Surfaces that Form an Enclosure

Generalized Cases

\[ \dot{Q}_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon \varepsilon A_1} + \frac{1}{A_1 F_{1R}} + \frac{1}{\varepsilon \varepsilon A_2} \left( \frac{1}{A_2 F_{2R}} \right)} \]