Unit Conversions with the Factor–Label Method

Three Simple Steps

Many engineering problems require unit conversions. For example, beam problems in Strength of Materials include beam lengths in feet (or meters), and beam depths and widths in inches (or millimeters). An equation containing dimensions in both feet and inches requires unit conversions.

The tradition in Physics is to convert everything to standard SI units, then write the equation with numbers (with no units), and hope that the units work out.

In Engineering disciplines, we use the Factor–Label Method of unit conversion to solve problems with mixed units. There are three steps to this method:

**Step 1** Write the problem algebraically so the desired quantity is to the left of the equals sign, and an algebraic equation is to the right of the equals sign.

**Step 2** Draw a horizontal line on the page, and enter numbers and units above and below the line as appropriate (i.e., in the numerator and denominator).

**Step 3** Draw a vertical line to show the separation between each unit conversion, and enter all unit conversions necessary to solve the problem. If the unit is raised to a power, then the conversion factor and unit must be raised to that power.

The units in the final answer must appear in the equation, and all other units must cancel.

Example #1

The area of a rectangle is \( A = bh \). Given a base \( b = 83 \) in. and a height \( h = 45 \) ft., calculate the area in square feet.

**Step 1** The algebraic equation does not need to be manipulated.

**Step 2** Draw a horizontal line. Enter 83 in. and 45 ft. in the numerator.

\[
A = \frac{83 \text{ in.}}{45 \text{ ft.}}
\]

**Step 3** We want to eliminate inches to obtain a final result in square feet. Therefore, put 12 inches in the denominator of the unit conversion, and 1 ft. in the numerator.

\[
A = \frac{83 \text{ in.} \times 45 \text{ ft.}}{12 \text{ in.}} = 311.25 \text{ ft.}^2
\]

Example #2

Deflection due to thermal expansion is \( \delta = \alpha L \Delta T \). Given a deflection \( \delta = 0.06 \) in., a length \( L = 8 \) ft., and a thermal expansion coefficient \( \alpha = 5 \times 10^{-6} \frac{\text{in.}}{\text{in.}^\circ \text{F}} \), calculate the change in temperature in degrees Fahrenheit.

**Step 1** Rewrite the equation algebraically to solve for \( \Delta T \).

\[
\Delta T = \frac{\delta}{\alpha L}
\]
**Step 2** Draw a horizontal line. Enter 0.06 in. in the numerator; enter 8 ft. and \(5 \times 10^{-6} \text{ in.} \text{in.}^\circ \text{F}\) in the denominator. Rewrite the equation so there are no fractional units in the numerator or the denominator, and it will be easier to cancel units.

\[
\Delta T = \frac{0.06 \text{ in.}}{8 \text{ ft.} \times 5 \times 10^{-6} \text{ in.} \text{in.}^\circ \text{F}} = \frac{0.06 \text{ in.}}{8 \text{ ft.} \times 5 \times 10^{-6} \text{in.}} \text{in.}^\circ \text{F}
\]

\[
\Delta T = \frac{0.06 \text{ in.}}{8 \text{ ft.} \times 5 \times 10^{-6} \text{in.}} \frac{\text{ft.}}{12 \text{ in.}} = 125^\circ \text{F}
\]

**Step 3** Convert feet to 12 inches so the length units to cancel, and the result is in degrees Fahrenheit.

\[
\Delta T = \frac{0.06 \text{ in.}}{8 \text{ ft.} \times 5 \times 10^{-6} \text{in.}} \frac{\text{ft.}}{12 \text{ in.}} = 125^\circ \text{F}
\]

**Example #3**

Stress is force divided by area: \(\sigma = \frac{P}{A}\). Given a force \(P = 7000 \text{ lb.}\) acting on an area \(A = 3 \text{ ft.}^2\), calculate the stress in units of pounds per square inch (psi).

**Step 1** The algebraic equation does not need to be manipulated.

**Step 2** Draw a horizontal line. Enter 7000 lb. in the numerator, and 3 ft. \(\times\) in the denominator.

\[
\sigma = \frac{P}{A} = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2}
\]

\[
\sigma = \frac{7000 \text{ lb.}}{3 \text{ ft.}^2} \left(\frac{\text{ft.}^2}{(12 \text{ in.})^2}\right) = 16.2 \text{ psi}
\]

**Example #4**

A tensile bar stretches an amount \(\delta = \frac{PL}{AE}\) where \(P\) is the applied load, \(L\) is the length of the bar, \(A\) is the cross-sectional area, and \(E\) is Young’s Modulus. If the bar has a circular cross section, then area \(A = \frac{\pi d^2}{4}\) where \(d\) is the diameter of the bar. Given a load of 30 kN, a length of 80 cm, a diameter of 6 mm, and a Young’s Modulus of 207 GPa, calculate the deflection in mm.

**Step 1** Combine the two equations to obtain a single algebraic equation.

**Step 2** Draw a horizontal line and enter the numbers and units.

**Step 3** The SI unit of stress or pressure is the pascal, where \(Pa = \frac{N}{m}\), so \(GPa = \frac{10^9 N}{m}\). Since \(kN = 10^3 N\), \(GPa = \frac{10^6 kN}{m}\).

Three conversion factors are needed: one to cancel GPa and kN; a second to cancel mm² and m²; and a third to put the final answer in mm.
### Symbols, Terminology, & Typical Units

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<th>Definition</th>
<th>Unit</th>
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<tr>
<td>( \alpha )</td>
<td>Thermal expansion coefficient</td>
<td>in./in./°F, m/m/°C</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Deflection due to thermal expansion; deflection due to tensile load</td>
<td>in., mm</td>
</tr>
<tr>
<td>( A )</td>
<td>Cross-sectional area</td>
<td>in.(^2), mm(^2)</td>
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<tr>
<td>( b )</td>
<td>Width of the base of a rectangle</td>
<td>in., mm</td>
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<tr>
<td>( d )</td>
<td>Diameter</td>
<td>in., mm</td>
</tr>
<tr>
<td>( h )</td>
<td>Height of a rectangle</td>
<td>in., mm</td>
</tr>
<tr>
<td>( E )</td>
<td>Young’s modulus (modulus of elasticity)</td>
<td>psi, ksi, GPa</td>
</tr>
<tr>
<td>( L )</td>
<td>Length</td>
<td>ft., in., m, mm</td>
</tr>
<tr>
<td>( P )</td>
<td>Point load</td>
<td>lb., kips, N, kN</td>
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<tr>
<td>( \Delta T )</td>
<td>Change in temperature</td>
<td>°F, °C</td>
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