Homework 4: Problem Solving Using Arrays

Assigned Feb. 22, 2002, Due March 8, 2002, post your source with sufficient documentation, a HTML page, and applet on your Web site for testing.

Background

Assume that you are working on a project and are asked to write, test, and document some Java program modules to help solving 3-D graphics and/or robotic problems.

We researched and found the book Robot Manipulators, Richard P. Paul, The MIT Press, 1982 defines some useful formulas for calculating Translation and Rotations that we need for solving this problem. After some meetings with team members, you are assigned the task to create and test some classes for these operations. The following descriptions were extracted form pages 11-24 of this book.

Vectors

A point vector can be represented as \( \mathbf{v} = ai + bj + ck \)

where \( i, j, \) and \( k \) are unit vectors along the \( x, y, \) and \( z \) coordinate axes, respectively, is represented in homogeneous coordinates as a column matrix

\[
\mathbf{v} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}
\]

where

\[
a = x/w \\
b = y/w \\
c = z/w
\]

Example 1:

Thus the vector \( \mathbf{v} = 3i + 4j + 5k \) can be represented as \( [3, 4, 5, 1]^T \) or as \( [6, 8, 10, 2]^T \) or again as \( [-30, -40, -50, -10]^T \), etc. The superscript \( T \) indicates the transpose of the row vector into a column vector. The vector at the origin, the null vector, is represented as \( [0, 0, 0, n]^T \) where \( n \) is any non-zero scale factor. The vector \( [0, 0, 0]^T \) is undefined.
**Planes**

A plane is represented as a row matrix (1 by 4)
\[ p = [a, b, c, d] \]

such that if a point \( v = [x, y, z, w]^T \), an 4 by 1 matrix, lies in a plane \( p \), the matrix product \((1 \times 4 \times 4 \times 1 = 1)\)
\[ pv = 0 \]

or in expanded form
\[ xa + yb + zc + wd = 0 \]

**Example 2:**

For example, a plane parallel to the (x,y) plane, one unit along the z-axis, is represented as
\[ p = [0, 0, 1, -1] \]
or \[ p = [0, 0, 2, -2] \]
or \[ p = [0, 0, -100, 100] \]

A point \( v = [10, 20, 1, 1] \) should lie in the plane
\[ [0, 0, 1, -1] [10, 20, 1, 1]^T = 0 \]
\[ [0, 0, 2, -2] [10, 20, 1, 1]^T = 0 \]
\[ [0, 0, -100, 100] [10, 20, 1, 1]^T = 0 \]

The point \( v = [0, 0, 2, 1] \) lies above the plane
\[ [0, 0, 2, -2] [0, 0, 2, 1]^T = 2 \]

The point \( v = [0, 0, 1, 1] \) lies below the plane
\[ [0, 0, 1, -1] [0, 0, 2, 1]^T = -1 \]

The plane \([0, 0, 0, 0]\) is undefined.

**Transformations**

A transformation of the space \( H \) is a 4x4 matrix and can represent translation, rotation, stretching, and perspective transformations. Given a point \( u \), its transformation \( v \) is represented by the matrix product
\[ v = Hu \]

**Translation Transformation**

The transformation \( H \) corresponding to a translation by a vector \( ai + bj + ck \) is
\[ \mathbf{H} = \text{Trans}(a, b, c) = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

Given a vector \( \mathbf{u} = [x, y, z, w]^T \) the transformed vector \( \mathbf{v} \) is given by

\[ \mathbf{v} = \begin{pmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x + aw \\ y + bw \\ z + cw \\ w \end{pmatrix} = \begin{pmatrix} x/w + a \\ y/w + b \\ z/w + c \\ 1 \end{pmatrix} \]

\textbf{Example 3}: Translate the vector \( 2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \) by \( 4\mathbf{i} - 3\mathbf{j} + 7\mathbf{k} \)

\[ \begin{bmatrix} 6 \\ 0 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} \]

\textbf{Rotation transformations}

The transformation corresponding to rotations about the x, y, or z axes by an angle \( \theta \) are

\[ \text{Rot}(x, \theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

\[ \text{Rot}(y, \theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \]
\[ \text{Rot}(z, \theta) = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

An Example: Given a point \( u = 7i + 3j + 2k \) what is the effect of rotating it 90 degree about the z axis to \( v \)?

With \( \sin 90 = 1 \) and \( \cos 90 = 0 \), the transformation \( \text{Rot}(x, \theta) \) is obtained as

\[ v = \text{Rot}(z,90) u \]

\[ v = \begin{bmatrix} -3 \\ 7 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \]

Example 4: Compute \( w = \text{Rot}(y,90) \text{Rot}(z,90)u \)

Where \( u = [7 \ 3 \ 2 \ 1]^T \) or

\[ \begin{bmatrix} 7 \\ 3 \\ 2 \\ 1 \end{bmatrix} \]

\[ \text{Rot}(y,90) \text{Rot}(z,90) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ \text{Rot}(y,90) \text{Rot}(z,90) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
Example 5: Calculate the following transformation $v = \text{Trans}(4, -3, 7) \text{Rot}(y, 90) \text{Rot}(z, 90) w$

$w = 7i + 3j + 2k$

$\text{TranA} = \text{Trans}(4, -3, 7) \text{Rot}(y, 90) \text{Rot}(z, 90) =$

\[
\begin{pmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & 7 \\
0 & 0 & 0 & 1
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

$w \text{ TransA} =
\begin{pmatrix}
6 \\
4 \\
10 \\
1
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 1 & 4 \\
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 7 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Homework Assignment: Given the discussion of matrix/vector operations, you are now assigned to solve Example 5 using Java Applet. The following class and methods are required. You should and can add additional methods for solving this problem:
- public class Robot
- Global arrays/matrixes within the Robot class
  - float f_rotx [4][4] - for storing Rot(x, theata) matrix
  - float f_roty [4][4] - for storing Rot(y, theata) matrix
  - float f_rotz [4][4] - for storing Rot(z, theata) matrix
  - float f_translate[4][4] - for storing Trans(a, b, c) matrix
- Methods for Robot class
  - void Rotx (float theta) method for setup f_rotx[4][4]
  - void Roty (float theta) method for setup f_roty[4][4]
  - void Rotz (float theta) method for setup f_roty[4][4]